

# Maths Programme of Study – Albany Academy

## Engage Springboard Pathway

MATHS	
PROGRAMME OF STUDY – ENGAGE SPRINGBOARD PATHWAY - ALBANY ACADEMY	
<b>Prior learning: KS3</b>	
<b>Working Mathematically</b>	<p>Through the mathematics content, pupils should be taught to:</p> <p><b>Develop fluency</b></p> <ul style="list-style-type: none"> <li>• consolidate their numerical and mathematical capability from key stage 2 and extend their understanding of the number system and place value to include decimals, fractions, powers and roots</li> <li>• select and use appropriate calculation strategies to solve increasingly complex problems</li> <li>• use algebra to generalise the structure of arithmetic, including to formulate mathematical relationships</li> <li>• substitute values in expressions, rearrange and simplify expressions, and solve equations</li> <li>• move freely between different numerical, algebraic, graphical and diagrammatic representations [for example, equivalent fractions, fractions and decimals, and equations and graphs]</li> <li>• develop algebraic and graphical fluency, including understanding linear and simple quadratic functions</li> <li>• use language and properties precisely to analyse numbers, algebraic expressions, 2-D and 3-D shapes, probability and statistics.</li> </ul> <p><b>Reason mathematically</b></p> <ul style="list-style-type: none"> <li>• extend their understanding of the number system; make connections between number relationships, and their algebraic and graphical representations</li> <li>• extend and formalise their knowledge of ratio and proportion in working with measures and geometry, and in formulating proportional relations algebraically</li> <li>• identify variables and express relations between variables algebraically and graphically</li> <li>• make and test conjectures about patterns and relationships; look for proofs or counter examples</li> <li>• begin to reason deductively in geometry, number and algebra, including using geometrical constructions</li> </ul>

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- interpret when the structure of a numerical problem requires additive, multiplicative or proportional reasoning
- explore what can and cannot be inferred in statistical and probabilistic settings, and begin to express their arguments formally.

#### **Solve problems**

- develop their use of formal mathematical knowledge to interpret and solve problems, including in financial mathematics
- begin to model situations mathematically and express the results using a range of formal mathematical representations
- select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems.

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Subject Content	Number
	<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> <li>• understand and use place value for decimals, measures and integers of any size</li> <li>• order positive and negative integers, decimals and fractions; use the number line as a model for ordering of the real numbers; use the symbols =, ≠, &lt;, ≥</li> <li>• use the concepts and vocabulary of prime numbers, factors (or divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation property</li> <li>• use the four operations, including formal written methods, applied to integers, decimals, proper and improper fractions, and mixed numbers, all both positive and negative</li> <li>• use conventional notation for the priority of operations, including brackets, powers, roots and reciprocals</li> <li>• recognise and use relationships between operations including inverse operations</li> <li>• use integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5 and distinguish between exact representations of roots and their decimal approximations</li> <li>• interpret and compare numbers in standard form <math>A \times 10^n</math> <math>1 \leq A &lt; 10</math> where n is a positive or negative integer or zero</li> <li>• work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and <math>\frac{2}{7}</math> or 0.375 and <math>\frac{8}{3}</math>)</li> <li>• define percentage as ‘number of parts per hundred’, interpret percentages and percentage changes as a fraction or a decimal, interpret these multiplicatively, express one quantity as a percentage of another, compare two quantities using percentages, and work with percentages greater than 100%</li> <li>• interpret fractions and percentages as operators</li> <li>• use standard units of mass, length, time, money and other measures, including with decimal quantities</li> <li>• round numbers and measures to an appropriate degree of accuracy [for example, to a number of decimal places or significant figures]</li> </ul>

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- use approximation through rounding to estimate answers and calculate possible resulting errors expressed using inequality notation  $a < x \leq b$
- use a calculator and other technologies to calculate results accurately and then interpret them appropriately
- appreciate the infinite nature of the sets of integers, real and rational numbers

#### **Algebra**

Pupils should be taught to:

- use and interpret algebraic notation, including:
  - $ab$  in place of  $a \times b$
  - $3y$  in place of  $y + y + y$  and  $3 \times y$
  - $a^2$  in place of  $a \times a$ ,  $a^3$  in place of  $a \times a \times a$ ;  $a^2 b$  in place of  $a \times a \times b$
  - $b \overline{a}$  in place of  $a \div b$
  - coefficients written as fractions rather than as decimals
  - brackets
- substitute numerical values into formulae and expressions, including scientific formulae
- understand and use the concepts and vocabulary of expressions, equations, inequalities, terms and factors
- simplify and manipulate algebraic expressions to maintain equivalence by: collecting like terms
  - multiplying a single term over a bracket
  - taking out common factors
  - expanding products of two or more binomials
- understand and use standard mathematical formulae; rearrange formulae to change the subject
- model situations or procedures by translating them into algebraic expressions or formulae and by using graphs
- use algebraic methods to solve linear equations in one variable (including all forms that require rearrangement)
- work with coordinates in all four quadrants

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- recognise, sketch and produce graphs of linear and quadratic functions of one variable with appropriate scaling, using equations in  $x$  and  $y$  and the Cartesian plane
- interpret mathematical relationships both algebraically and graphically
- reduce a given linear equation in two variables to the standard form  $y = mx + c$ ; calculate and interpret gradients and intercepts of graphs of such linear equations numerically, graphically and algebraically
- use linear and quadratic graphs to estimate values of  $y$  for given values of  $x$  and vice versa and to find approximate solutions of simultaneous linear equations
- find approximate solutions to contextual problems from given graphs of a variety of functions, including piece-wise linear, exponential and reciprocal graphs
- generate terms of a sequence from either a term-to-term or a position-to-term rule
- recognise arithmetic sequences and find the  $n$ th term
- recognise geometric sequences and appreciate other sequences that arise

#### **Ratio, proportion and rates of change**

Pupils should be taught to:

- change freely between related standard units [for example time, length, area, volume/capacity, mass]
- use scale factors, scale diagrams and maps
- express one quantity as a fraction of another, where the fraction is less than 1 and greater than 1
- use ratio notation, including reduction to simplest form
- divide a given quantity into two parts in a given part:part or part:whole ratio; express the division of a quantity into two parts as a ratio
- understand that a multiplicative relationship between two quantities can be expressed as a ratio or a fraction
- relate the language of ratios and the associated calculations to the arithmetic of fractions and to linear functions

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- solve problems involving percentage change, including: percentage increase, decrease and original value problems and simple interest in financial mathematics
- solve problems involving direct and inverse proportion, including graphical and algebraic representations
- use compound units such as speed, unit pricing and density to solve problems.

#### **Geometry and measures**

Pupils should be taught to:

- derive and apply formulae to calculate and solve problems involving: perimeter and area of triangles, parallelograms, trapezia, volume of cuboids (including cubes) and other prisms (including cylinders)
- calculate and solve problems involving: perimeters of 2-D shapes (including circles), areas of circles and composite shapes
- draw and measure line segments and angles in geometric figures, including interpreting scale drawings
- derive and use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle); recognise and use the perpendicular distance from a point to a line as the shortest distance to the line
- describe, sketch and draw using conventional terms and notations: points, lines, parallel lines, perpendicular lines, right angles, regular polygons, and other polygons that are reflectively and rotationally symmetric
- use the standard conventions for labelling the sides and angles of triangle ABC, and know and use the criteria for congruence of triangles  $\square$  derive and illustrate properties of triangles, quadrilaterals, circles, and other plane figures [for example, equal lengths and angles] using appropriate language and technologies
- identify properties of, and describe the results of, translations, rotations and reflections applied to given figures
- identify and construct congruent triangles, and construct similar shapes by enlargement, with and without coordinate grids
- apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles

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- understand and use the relationship between parallel lines and alternate and corresponding angles
- derive and use the sum of angles in a triangle and use it to deduce the angle sum in any polygon, and to derive properties of regular polygons
- apply angle facts, triangle congruence, similarity and properties of quadrilaterals to derive results about angles and sides, including Pythagoras' Theorem, and use known results to obtain simple proofs
- use Pythagoras' Theorem and trigonometric ratios in similar triangles to solve problems involving right-angled triangles
- use the properties of faces, surfaces, edges and vertices of cubes, cuboids, prisms, cylinders, pyramids, cones and spheres to solve problems in 3-D  $\square$  interpret mathematical relationships both algebraically and geometrically

#### **Probability**

Pupils should be taught to:

- record, describe and analyse the frequency of outcomes of simple probability experiments involving randomness, fairness, equally and unequally likely outcomes, using appropriate language and the 0-1 probability scale
- understand that the probabilities of all possible outcomes sum to 1
- enumerate sets and unions/intersections of sets systematically, using tables, grids and Venn diagrams
- generate theoretical sample spaces for single and combined events with equally likely, mutually exclusive outcomes and use these to calculate theoretical probabilities.

#### **Statistics**

Pupils should be taught to:

- describe, interpret and compare observed distributions of a single variable through: appropriate graphical representation involving discrete, continuous and grouped data; and appropriate measures of central tendency (mean, mode, median) and spread (range, consideration of outliers)

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	<ul style="list-style-type: none"> <li>• construct and interpret appropriate tables, charts, and diagrams, including frequency tables, bar charts, pie charts, and pictograms for categorical data, and vertical line (or bar) charts for ungrouped and grouped numerical data</li> <li>• describe simple mathematical relationships between two variables (bivariate data) in observational and experimental contexts and illustrate using scatter graphs</li> </ul>
	<p>It is important to note that:</p> <ul style="list-style-type: none"> <li>- the mathematical content that should be taught to all pupils, in standard type; and</li> <li>- together, the mathematical content set out in the key stage 3 and key stage 4 programmes of study covers the full range of material contained in the GCSE Mathematics qualification.</li> </ul>
<p><b>Working Mathematically</b></p>	<p>Through the mathematics content, pupils should be taught to:</p> <p><b>Develop fluency</b></p>

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- consolidate their numerical and mathematical capability from key stage 3 and extend their understanding of the number system to include powers, roots {and fractional indices}
- select and use appropriate calculation strategies to solve increasingly complex problems, including exact calculations involving multiples of  $\pi$  {and surds}, use of standard form and application and interpretation of limits of accuracy
- consolidate their algebraic capability from key stage 3 and extend their understanding of algebraic simplification and manipulation to include quadratic expressions, {and expressions involving surds and algebraic fractions}
- extend fluency with expressions and equations from key stage 3, to include quadratic equations, simultaneous equations and inequalities
- move freely between different numerical, algebraic, graphical and diagrammatic representations, including of linear, quadratic, reciprocal, {exponential and trigonometric} functions
- use mathematical language and properties precisely.

#### **Reason mathematically**

- extend and formalise their knowledge of ratio and proportion, including trigonometric ratios, in working with measures and geometry, and in working with proportional relations algebraically and graphically
- extend their ability to identify variables and express relations between variables algebraically and graphically
- make and test conjectures about the generalisations that underlie patterns and relationships; look for proofs or counter-examples; begin to use algebra to support and construct arguments {and proofs}
- reason deductively in geometry, number and algebra, including using geometrical constructions
- interpret when the structure of a numerical problem requires additive, multiplicative or proportional reasoning
- explore what can and cannot be inferred in statistical and probabilistic settings, and express their arguments formally
- assess the validity of an argument and the accuracy of a given way of presenting information.

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	<p><b>Solve problems</b></p> <ul style="list-style-type: none"> <li>• develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems</li> <li>• develop their use of formal mathematical knowledge to interpret and solve problems, including in financial contexts</li> <li>• make and use connections between different parts of mathematics to solve problems</li> <li>• model situations mathematically and express the results using a range of formal mathematical representations, reflecting on how their solutions may have been affected by any modelling assumptions</li> <li>• select appropriate concepts, methods and techniques to apply to unfamiliar and non</li> </ul>
<b>Subject Content</b>	<p><b>Number</b></p> <p>In addition to consolidating subject content from key stage 3, pupils should be taught to:</p> <ul style="list-style-type: none"> <li>• apply systematic listing strategies,</li> <li>• calculate with roots, and with integer indices</li> <li>• calculate exactly with fractions, and multiples of <math>\pi</math>;</li> <li>• calculate with numbers in standard form <math>A \times 10^n</math>, where <math>1 \leq A &lt; 10</math> and <math>n</math> is an integer</li> <li>• identify and work with fractions in ratio problems</li> <li>• apply and interpret limits of accuracy when rounding or truncating, {including upper and lower bounds}.</li> </ul> <p><b>Algebra</b></p> <p>In addition to consolidating subject content from key stage 3, pupils should be taught to:</p> <ul style="list-style-type: none"> <li>• simplify and manipulate algebraic expressions (including those involving surds by:</li> <li>• factorising quadratic expressions, including the difference of two squares;</li> <li>• simplifying expressions involving sums, products and powers, including the laws of indices</li> </ul>

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- know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments
- where appropriate, interpret simple expressions as functions with inputs and outputs;
- use the form  $y = mx + c$  to identify parallel lines; find the equation of the line through two given points, or through one point with a given gradient
- identify and interpret roots, intercepts and turning points of quadratic functions graphically; deduce roots algebraically
- recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function  $y = \frac{1}{x}$ ,  $y = \cos x$  with  $x \neq 0$ , and  $y = \tan x$  for angles of any size}
- plot and interpret graphs (including reciprocal graphs and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration
- solve quadratic equations {including those that require rearrangement} algebraically by factorising, {by completing the square and by using the quadratic formula}; find approximate solutions using a graph
- solve two simultaneous equations in two variables algebraically; find approximate solutions using a graph
- translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution
- solve linear inequalities in one variable,; represent the solution set on a number line
- recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions, Fibonacci type sequences, quadratic sequences, and simple geometric progressions ( $r^n$  where  $n$  is an integer, and  $r$  is a positive rational number)
- deduce expressions to calculate the  $n$ th term of linear sequences.

#### **Ratio, proportion and rates of change**

In addition to consolidating subject content from key stage 3, pupils should be taught to:

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- compare lengths, areas and volumes using ratio notation and/or scale factors; make links to similarity (including trigonometric ratios)
- convert between related compound units (speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts
- understand that  $X$  is inversely proportional to  $Y$  is equivalent to  $X$  is proportional to  $1/Y$ ; interpret equations that describe direct and inverse proportion
- interpret the gradient of a straight-line graph as a rate of change; recognise and interpret graphs that illustrate direct and inverse proportion
- set up, solve and interpret the answers in growth and decay problems, including compound interest {and work with general iterative processes}.

#### **Geometry and measures**

In addition to consolidating subject content from key stage 3, pupils should be taught to:

- interpret and use fractional scale factors for enlargements
- identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
- construct and interpret plans and elevations of 3D shapes
- interpret and use bearings
- calculate arc lengths, angles and areas of sectors of circles
- calculate surface areas and volumes of spheres, pyramids, cones and composite solids
- apply the concepts of congruence and similarity, including the relationships between lengths, in similar figures
- apply Pythagoras' Theorem and trigonometric ratios to find angles and lengths in right-angled triangles {and, where possible, general triangles} in two dimensional figures
- know the exact values of sin cos and tan

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- describe translations as 2D vectors
- apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors;

#### Probability

In addition to consolidating subject content from key stage 3, pupils should be taught to:

- apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one
- use a probability model to predict the outcomes of future experiments; understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size
- calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions

#### Statistics

In addition to consolidating subject content from key stage 3, pupils should be taught to:

- infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling
- interpret and construct tables and line graphs for time series data
- interpret, analyse and compare the distributions of data sets from univariate empirical distributions through:
- appropriate graphical representation involving discrete, continuous and grouped data, {including box plots}
- appropriate measures of central tendency (including modal class) and spread
- apply statistics to describe a population
- use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation; draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing.



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<b>Subsequent Learning</b>	<p><i>Post-16+</i></p> <p>This programme of study is designed to allow progression within other GCSE courses and to support continuation and potential entry for GCSE or Level 1 or 2 qualifications at Dual Placement schools. This programme of study could lead to supporting with skills required for a range of Level 2 and Level 3 qualifications or other vocational qualifications. It should support students to transition to adult life with transferrable skills.</p>
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